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Flavor symmetry breaking and strangeness in the nucleon

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Abstract. We suggest that breaking of $SU(3)$ flavor symmetry mainly resides in the baryon wave functions while the charge operators have no (or only small) explicit symmetry-breaking components. We utilize the collective coordinate approach to chiral soliton models to support this picture. In particular we compute the g_A/g_V ratios for hyperon beta-decay and the strangeness contribution to the nucleon axial current matrix elements and analyze their variation with increasing flavor symmetry breaking.

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1 Introduction and motivation

There has been much interest in the strangeness content of the nucleon ever since the analysis of the DIS data [1] suggested a large (negative) polarization of strange quarks in the nucleon [2,3], $\triangle S_N \approx -0.15$. This surprising result particularly relies on the assumption of flavor covariance for the axial current matrix elements of the $\frac{1}{2}^+$ baryons. This assumption originates from the feature that the Cabibbo scheme [4], that utilizes the $F\&D$ parameterization for the flavor changing axial charges, works unexpectedly well [5] as the comparison in table 1 shows.

Here we will investigate in how far this agreement justifies to carry over flavor covariance to strangeness-conserving axial current matrix elements in order to disentangle the various quark flavor components of the nucleon axial current matrix element. This investigation requires baryon axial current matrix elements as functions of the (effective) strength of flavor symmetry breaking. This can be achieved within the three-flavor version of the Skyrme model (and generalizations thereof) in which baryons emerge as solitons. In such models baryon states are constructed by quantizing the large amplitude fluctuations about the soliton and constructing exact eigenstates in the presence of symmetry breaking. We focus on a picture where symmetry breaking mainly resides in the baryon wave functions, including important contributions which would be missed in a first-order treatment. In contrast, we assume that the current operators, from which the charges are computed, are dominated by flavor covariant components. In a first step we do not specify the model Lagrangian but adjust the prefactors of the few possible flavor covariant components of the axial current operator to observables in hyperon beta-decay and analyze their matrix elements as functions of flavor symmetry breaking. We also present results obtained from a realistic vector meson soliton model that supports the suggested picture. Details omitted here may be traced from ref [8].

Table 1. The empirical values for the g_A/g_V ratios of hyperon beta-decays [6], see also [5]. For $\Sigma \to \Lambda$ only g_A is given. Also the flavor symmetric predictions are presented using $F = 0.459$ and $D = 0.799$. Analytic expressions which relate these parameters to the g_A/g_V ratios may *e.g.* be found in table I of [7].

	$\Lambda \to p$	$\Sigma \to n$	$\Xi \to \Lambda$
Empirical	0.718 ± 0.015	0.340 ± 0.017	0.25 ± 0.05
$F\&D$	0.725 ± 0.009	0.339 ± 0.026	0.19 ± 0.02
	$\Xi \to \Sigma$	$\Sigma \to \Lambda$	
Empirical	1.287 ± 0.158	0.61 ± 0.02	
$F\&D$	$1.258 = g_A$	0.65 ± 0.01	

2 Symmetry breaking in the baryon wave functions

The collective coordinates A that parameterize the large amplitude fluctuations off the soliton are introduced via

$$U(\boldsymbol{r},t) = A(t)U_0(\boldsymbol{r})A^\dagger(t)\,, \qquad A(t) \in SU(3)\,. \quad (1)$$

$U_0(\boldsymbol{r})$ describes the soliton embedded in the isospin subgroup. A prototype model Lagrangian for $U(\boldsymbol{r},t)$ consists of the Skyrme model supplemented by the Wess-Zumino-Witten term and suitable symmetry-breaking pieces. We

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parameterize the collective coordinates by "Euler angles"

$$A = D_2(\hat{I})\,\mathrm{e}^{-i\nu\lambda_4} D_2(\hat{R})\,\mathrm{e}^{-i(\rho/\sqrt{3})\lambda_8}\,. \qquad (2)$$

Here D_2 denote the rotation matrices for rotations in isospace $(\hat{I})$ and coordinate space $(\hat{R})$. Substituting (1) into the model Lagrangian yields upon canonical quantization the Hamiltonian for the collective coordinates A:

$$H = H_\mathrm{s} + \tfrac{3}{4}\,\gamma\sin^2\nu\,. \qquad (3)$$

The symmetric piece of this collective Hamiltonian only contains Casimir operators and may be expressed in terms of the $SU(3)$-right generators R_a $(a = 1, \ldots, 8)$:

$$H_\mathrm{s} = M_\mathrm{cl} + \frac{1}{2\alpha^2}\sum_{i=1}^{3} R_i^2 + \frac{1}{2\beta^2}\sum_{\alpha=4}^{7} R_\alpha^2\,. \qquad (4)$$

$M_\mathrm{cl}, \alpha^2, \beta^2$ and γ are functionals of the soliton, $U_0(\boldsymbol{r})$. The generators R_a can be expressed in terms of derivatives with respect to the "Euler angles". The essential feature of the parameterization (2) is that the flavor-symmetry-breaking part of the full Hamiltonian (3) only depends on the flavor-changing angle ν. Therefore the eigenvalue problem $H\Psi = \epsilon\Psi$ reduces to ordinary second-order differential equations for isoscalar functions which only depend on ν [9]. Solely the product $\omega^2 = \frac{3}{2}\gamma\beta^2$ appears in these differential equations as the effective strength of symmetry breaking on which the eigenfunctions of H depend parametrically. A value in the range $5 \lesssim \omega^2 \lesssim 8$ is required to obtain reasonable agreement with the empirical mass differences for the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons [10]. Such large a value for ω^2 is without reach of a perturbation expansion as the resulting baryon wave functions exhibit strong distortion from flavor covariance.

3 Charge operators

In the soliton description the effect of the derivative-type symmetry-breaking terms is mainly indirect. They provide the splitting between the various decay constants and thus increase γ since it is proportional to $f_K^2 m_K^2 - f_\pi^2 m_\pi^2 \approx 1.5 f_\pi^2(m_K^2 - m_\pi^2)$. Otherwise, the derivative-type symmetry-breaking terms are negligible. Whence symmetry-breaking terms can be omitted in the current operators and the non-singlet axial charge operator is parameterized as $(a = 1, \ldots, 8,\ i = 1, 2, 3)$

$$\int \mathrm{d}^3 r A_i^{(a)} = c_1 D_{ai} - c_2 D_{a8} R_i + c_3 \sum_{\alpha,\beta=4}^{7} d_{i\alpha\beta} D_{a\alpha} R_\beta\,, \quad (5)$$

where $D_{ab} = \frac{1}{2}\mathrm{tr}\left(\lambda_a A \lambda_b A^\dagger\right)$. For $\omega^2 \to \infty$ (infinitely heavy *strange* degrees of freedom) the strangeness contribution to the nucleon axial charge should vanish. Noting that $\langle N|D_{83}|N\rangle \to 0$ and $\langle N|\sum_{\alpha,\beta=4}^{7} d_{3\alpha\beta} D_{8\alpha} R_\beta|N\rangle \to 0$ while $\langle N|D_{88}|N\rangle \to 1$ for $\omega^2 \to \infty$, we demand

$$\int \mathrm{d}^3 r A_i^{(0)} = -2\sqrt{3}c_2 R_i\,, \quad i = 1, 2, 3 \qquad (6)$$

for the axial singlet current because it leads to the strangeness projection, $A_i^{(s)} = (A_i^{(0)} - 2\sqrt{3}A_i^{(8)})/3$ that vanishes for $\omega^2 \to \infty$. Actually all model calculations in the literature [11,12] are consistent with this relation between singlet and octet currents. The singlet current matrix element, $\triangle\Sigma_B = \sqrt{3}c_2$, is the quark spin contribution to the spin of the considered baryon, B. It is well known that the empirical value for the nucleon matrix element, $\triangle\Sigma_N \approx 0.20 \pm 0.10$ [3] is insensitive to the strength of flavor symmetry breaking [13]. This suggests to adjust c_2 accordingly. In order to completely describe the hyperon beta-decays we also demand matrix elements of the vector charges. These are obtained from the operator

$$\int \mathrm{d}^3 r V_0^{(a)} = \sum_{b=1}^{8} D_{ab} R_b = L_a, \qquad (7)$$

which introduces the $SU(3)$-left generators L_a.

The values for g_A and g_V (only g_A for $\Sigma^+ \to \Lambda e^+ \nu_e$) are obtained from the matrix elements of the operators in eqs (5) and (7), respectively, sandwiched between the eigenstates of the full Hamiltonian (3). We still have to specify c_1 and c_3. We determine these two parameters such that nucleon axial charge, g_A and the g_A/g_V ratio for $\Lambda \to p e^- \bar{\nu}_e$ are reproduced[1] at a prescribed strength of flavor symmetry breaking, $\omega_\mathrm{fix}^2 = 6.0$. Then we are not only left with predictions for the other decay parameters but we can in particular study the variation with symmetry breaking. This is shown in fig. 1. The dependence on flavor symmetry breaking is very moderate[2] and the results can be viewed as reasonably agreeing with the empirical data, cf. table 1. The observed independence of ω^2 shows that these predictions are not sensitive to the choice of ω_fix^2. The two transitions, $n \to p$ and $\Lambda \to p$, which are not shown in fig. 1, exhibit a similar negligible dependence on ω^2. We therefore have a two parameter (c_1 and c_3, c_2 is fixed from $\Delta\Sigma_N$) fit of the hyperon beta-decays. Comparing the results in fig. 1 with the data in table 1 we see that the present calculation using the strongly distorted wave functions agrees equally well with the empirical data as the flavor-symmetric $F\&D$ fit. On the other hand, the strangeness contribution to the nucleon axial current matrix element reduces from $\triangle S_N \approx -0.13$ in the symmetric treatment to $\triangle S_N \approx -0.07$ in the realistic case.

4 Model calculation

We consider a realistic soliton model containing pseudoscalar and vector meson fields. It has been established for two flavors in ref. [14] and been extended to three flavors in ref. [11] where it has been shown to fairly well describe the parameters of hyperon beta-decay (cf. table 4 in ref. [11]). The model Lagrangian contains terms which involve the

[1] In this section we will not address the problem of the too small model prediction for g_A.

[2] However, the individual matrix elements entering the ratios g_A/g_V vary strongly with ω^2 [8].

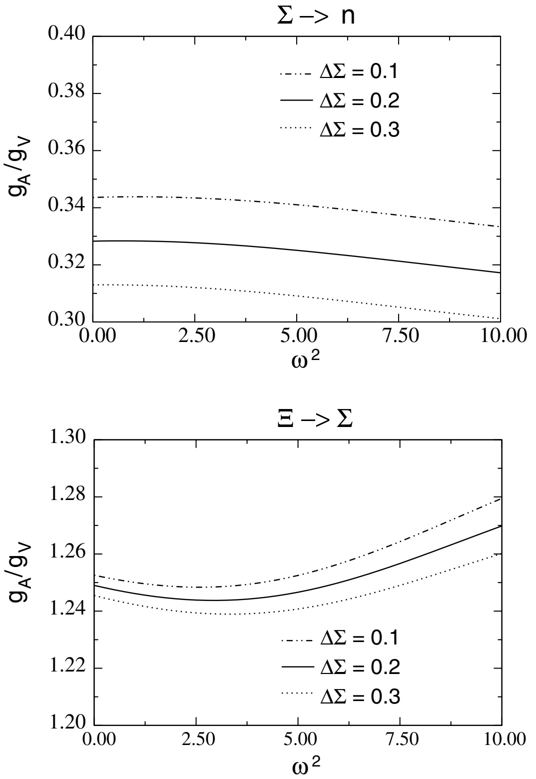

Fig. 1. The predicted decay parameters for the hyperon beta-decays using $\omega_{\rm fix}^2 = 6.0$. The errors originating from those in $\Delta\Sigma_N$ are indicated.

Levi-Cevita tensor $\epsilon_{\mu\nu\rho\sigma}$, to accommodate processes like $\omega \to 3\pi$ [15]. Such terms contribute to c_2 and c_3. A minimal set of symmetry-breaking terms is included [16] to account for different masses and decay constants. They add symmetry-breaking pieces to the axial charge operator,

$$\begin{aligned}\delta A_i^{(a)} &= c_4 D_{a8} D_{8i} + c_5 \sum_{\alpha,\beta=4}^{7} d_{i\alpha\beta} D_{a\alpha} D_{8\beta} \\ &\quad + c_6 D_{ai}(D_{88}-1)\,, \\ \delta A_i^{(0)} &= 2\sqrt{3}\, c_4 D_{8i}\,.\end{aligned}$$

Unfortunately the model parameters cannot be completely determined in the meson sector [14]. We use the remaining freedom to accommodate baryon properties in three different ways as shown in table 2. The set denoted by "b.f." refers to a best fit to the baryon spectrum. It predicts the axial charge somewhat on the low side, $g_A = 0.88$. The entry "mag. mom." labels parameters that yield magnetic moments close to the empirical data (with $g_A = 0.98$) and finally the set labeled "g_A" reproduces the axial charge of the nucleon [11]. We observe that in particular the strangeness projection of the nucleon axial current is very small and depends only mildly on the model parameters. This confirms the above conclusion from the general structure of the axial current matrix elements that the strangeness admixture in the nucleon is significantly smaller than an analysis based on flavor covariance suggests. Also the predictions for the axial properties of the Λ hyperon are quite insensitive to the model parameters. Sizable polarizations of the *up* and *down* quarks in the Λ are predicted; comparable to those obtained from the $SU(3)$ analysis [17] of the available data.

Table 2. Quark spin content of the nucleon and the Λ in the realistic vector meson model. Three sets of model parameters are considered, see text.

	N			
	ΔU	ΔD	ΔS	$\Delta\Sigma$
b.f.	0.603	−0.279	−0.034	0.291
mag. mom.	0.636	−0.341	−0.030	0.265
g_A	0.748	−0.476	−0.016	0.256

	Λ		
	$\Delta U = \Delta D$	ΔS	$\Delta\Sigma$
b.f.	−0.155	0.567	0.256
mag. mom.	−0.166	0.570	0.238
g_A	−0.164	0.562	0.233

5 Conclusions

We have suggested a picture for the axial charges of the low-lying $\frac{1}{2}^+$ baryons which manages to reasonably reproduce the empirical data without introducing (significant) flavor-symmetry-breaking components in the corresponding operators. Rather, a sizable symmetry breaking, as demanded by the baryon spectrum, resides almost completely in the baryon wave functions. In this picture the empirical data for hyperon beta-decay are as reasonably reproduced as in the Cabibbo scheme. We emphasize that the present picture is not a re-application of the Cabibbo scheme since here the "octet" baryon wave functions have significant admixture of higher-dimensional representations. Especially, when compared with the flavor-covariant treatment, the present approach predicts a sizable suppression of strangeness in the nucleon.

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